J. Sargeant Reynolds Community College
Course Content Summary

Course Prefix and Number: MTH 282 **Credits:** 3

Course Title: Mathematical Reasoning: Introduction to Higher Mathematics

Course Description:
Introduces topics in upper-level mathematics courses, such as mathematical reasoning and proofs, set theory, abstract algebra, and abstract analysis. Covers logic and methods of proof; set theory and cardinality; deductive reasoning and axiomatic method; introduction to groups, rings, and fields; construction of real numbers, and basic combinatorics. Prerequisites: MTH 164, MTH 166 or above or permission of instructor. Lecture 3 hours per week.

Course Prerequisites:
Satisfactory completion of MTH 166 or higher level mathematics course or instructor's permission

General Course Purpose:
This course serves as a transition course for students transferring from two-year to four-year colleges and universities to major in STEM fields of study (Science, Technology, Engineering, and Mathematics). Focusing on the types of skills and the level of abstraction required to be successful in 3rd and 4th year mathematics courses, this course will transfer as an equivalent course to Virginia Commonwealth University as MATH 300 – Introduction to Mathematical Reasoning.

Course Objectives:
Upon completing the course, the student will be able to:
1. Apply the appropriate techniques to construct proofs of mathematical statements in a variety of contexts utilizing various methods of proof: direct and contrapositive proof, reduction to contradiction, existence and uniqueness, and disproof.
2. Demonstrate sufficient knowledge and reasoning skills by proving statements involving relations (equivalence, partitions, and closure), functions (injective, bijective, inverse), divisibility and congruence, and mathematical induction and recursion.
3. Classify and distinguish various finite and infinite sets; apply the elements of basic set theory to construct proofs of statements involving the concepts of set membership, subsets, and cardinality.
4. Identify and relate the various components of the axiomatic method to analyze the concepts of consistency, independence, and completeness of an axiom system.
5. Apply relevant properties of groups, rings, and fields to analyze given sets and operations; construct proofs of basic statements regarding groups, rings, and fields.
6. Utilize relevant properties of natural, integer, rational, real, and complex numbers to analyze concepts and solve problems involving the construction of the fields of real and complex numbers.
7. Solve basic combinatorial problems relating to counting principles, permutations and combinations, binomial coefficients, inclusion-exclusion principle, and basic discrete probability.

Major Topics to be Included:
1. Logic
   a. Sentential logic
   b. Conditional and biconditional; material implication
   c. Premises and conclusions; argument forms

   d. Deduction and induction
   e. Truth and validity
   f. Categorical propositions and categorical syllogisms
   g. Elements of symbolic logic
   h. Rules of inference and rules of replacement
   i. Formal proof of validity
   j. Self-contradiction and inconsistency
2. Methods of Proof
   a. Direct proof
   b. Contrapositive proof
   c. Reduction to contradiction proof
   d. Existence and uniqueness proof
   e. Disproof
   f. Relations (equivalence, partitions, closure)
   g. Functions (injective, bijective, inverse)
   h. Divisibility and congruence
   i. Principle of mathematical induction and recursion
3. Set Theory and Cardinality:
   a. Basics (naive set theory)
   b. Cartesian products, power sets
   c. Finite and infinite sets and dedekind infinite sets
   d. Countable and uncountable sets; cardinality
   e. Difference between “naive” and axiomatic set theory
4. Deductive Reasoning and Axiomatic Method:
   a. Definitions
   b. Axioms (postulates)
   c. Undefined terms
   d. Proving of theorems (formally derivable statements)
   e. Consistency, independence, and completeness of axiom system
5. Introduction to Abstract Algebra:
   a. Definition and properties of a group
   b. Definition and properties of a ring
   c. Definition and properties of a field
6. Introduction to the Construction of Continuum:
   a. Arithmetic of natural numbers; induction
   b. Peano’s axioms and properties of **N**
   c. Integers as a ring
   d. Fractions as ordered pairs; rational number as an equivalence class
   e. Rational numbers as an ordered field
   f. Real numbers as a complete ordered field
   g. Complex number as an ordered pair
   h. Complex numbers as (unordered) field
7. Basic Combinatorics:
   a. Sets and sequences
   b. Counting principles
   c. Inclusion-exclusion principle
   d. Permutations and combinations
   e. Binomial coefficients
   f. Discrete probability

Effective Date of Course Content Summary: September 1, 2012